LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2025**

UMT 4601 – COMBINATORICS

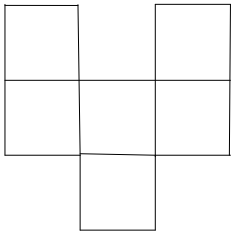
Date: 08-05-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions - **(10 x 1 = 10)**
1.	**Answer the following**
a)	Find the solution to the recurrence relation $a_n = 2a_{n-1}, n > 1$ with initial condition $a_1 = 2$.
b)	How many ATM PINs can be generated by a bank?
c)	Stretch a 3×4 Latin rectangle.
d)	Write all possible derangement of the numbers 1, 2, 3, 4.
e)	Define rook polynomial.
2.	**Fill in the blanks**
a)	The fourth term of the recurrence relation $a_n = a_{n-1}^2, n > 1$ and $a_1 = 2$ is _____ .
b)	The total number of 3-digit numbers is _____.
c)	A perfect matching is only possible if the graph has an _____ number of vertices.
d)	The associated quadratic equation of the recurrence relation $a_n = 3a_{n-1} + 2a_{n-2}$ is _____.
e)	Identify the rook polynomial of the following board. 
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**MCQ**
a)	The coefficient of x^6 in the expansion $(1-x)^{-5}$ is (i) 200 (ii) 202 (iii) 205 (iv) 210
b)	The number of 2-digit numbers without repetition of digits is (i) 81 (ii) 90 (iii) 99 (iv) 100
c)	The total number of assignments in a 4 × 4 assignment problem is (i) 24 (ii) 16 (iii) 64 (iv) 256
d)	The generating function for the sequence $1, 3, 9, 27 \ldots$ is

	(i) $(1-x)^{-1}$ for $\|x\|<1$ (ii) $(1+x)^{-1}$ for $\|x\|<1$ (iii) $(1-3x)^{-1}$ for $\|x\|<\frac{1}{3}$ (iv) $(1+3x)^{-1}$ $\|x\|<\frac{1}{3}$
e)	If $\|A\|$ is the number of elements of the set A, then $\|A \cup B\|$ is equal to (i) $\|A\|+\|B\|$ (ii) $\|A\|+\|B\|-\|A\|\|B\|$ (iii) $\|A\|+\|B\|-\|A \cap B\|$ (iv) $\|A\|\|B\|$
4.	**True or False**
a)	The expression $f(3,10)$ represents the number of ways of colouring 3 indistinguishable golf balls with 10 colours.
b)	$c(n,r)n = p(n,r)$ holds always.
c)	Suppose there are five jobs available and the set $\{S_i : 1 \le i \le 5\}$ denotes the applicants suited for i^{th} job. If $S_1=\{A,B,C\}$, $S_2=\{D,E\}$, $S_3=\{D\}$, $S_4=\{E\}$, $S_5=\{A,E\}$, then all jobs can be assigned to an applicant.
d)	The generating function for the sequence 1, 0, 1, 0, 1, 0 … is $(1-x)^{-1}$.
e)	The number of permutations of n symbols in which no symbol is in a forbidden position is $\sum_{k=0}^{n}(-1)^k(n+k)!\,r_k$ where r_k is the number of ways of placing k non-taking rooks on the board of forbidden positions.
	SECTION B - K3 (CO2)
Answer any TWO of the following	**(2 x 10 = 20)**
5.	A football league consisting of n teams, where each team competes against every other team twice. If the total number of games played is $2c$, where c represents the number of ways to choose 2 teams from n teams, then derive the formula $c=\frac{n(n-1)}{2}$. Additionally, compute the total number of matches played in a league with 25 teams.
6.	Prove that $\exp(x+y)=\exp(x)\exp(y)$, by using binomial theorem.
7.	Consider a set S with mn objects. Determine the number of different ways in which S can be partitioned into n sets of m elements. Hence find the number of ways in which 20 towns can be put into 5 groups of 4.
8.	Develop a recurrence relation to find the number of distinct simple rooted trees with n vertices.
	SECTION C – K4 (CO3)
Answer any TWO of the following	**(2 x 10 = 20)**
9.	How many 9-digit numbers can be formed by using each of the digits 1, 2, …, 9 exactly once? How many of these are bigger than 600 000 000?
10.	State Landau's theorem and establish its proof. Apply the theorem, to evaluate whether the sequence (0, 1, 1, 4, 4) qualifies as a valid score sequence.
11.	By taking $f(x)=\sum_{n=0}^{\infty}a_nx^n$, solve the recurrence relation $a_n=6a_{n-1}-9a_{n-2}$ subject to the initial conditions $a_0=2, a_1=6$.

12. Determine the rook polynomial of the given chess board:



SECTION D – K5 (CO4)

Answer any ONE of the following **(1 x 20 = 20)**

13. The function $g(n,k)$ represents the number of ways to place k indistinguishable lions in n cages, ensuring that no two lions are in consecutive cages and each cage holds at most one lion. Verify the following statements:

(i) $g(2k-1,k)=1$.
(ii) $g(n,k)=0, if\ n<2k-1$.
(iii) $g(n,1)=n$.
(iv) $g(n,k)=g(n-2,k-1)+g(n-1,k)$ if $k\geq 2$.
(v) $g(6,3)=4$.
(vi) $g(2k,k)=g(2k-2,k-1)+1$.
(vii) $g(2k,k)=k+1$.

14. (a) Justify with a proof that $\binom{n}{r}=\binom{n-1}{r-1}+\binom{n-1}{r}$.

(b) Find the rook polynomial for a 4×4 chess board.

SECTION E – K6 (CO5)

Answer any ONE of the following **(1 x 20 = 20)**

15. (i) 30 women including Miss. India enter a Miss World competition. The first 6 places are announced. How many different announcements are possible? How many if Miss. India is assured of a place in the first six?
(ii) A sports magazine decides to publish articles on all 10 IPL cricket teams, one team per week for 10 weeks. In how many ways can this be done if the teams CSK and MI must be featured on consecutive weeks?

16. (i) Formulate an optimal assignment strategy for four men traveling from towns A,B,C,D to towns a,b,c,d with one man assigned to each destination and the distance table is given by.

	A	B	C	D
a	5	7	15	12
b	8	3	9	10
c	4	14	2	5
d	6	3	1	14

(ii) Design an explicit formula for Fibonacci sequence of numbers. (10 + 10)

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